Polynomial Long Division

When you are required to factorise or solve a polynomial equation with a degree greater than two, you need to use polynomial division (or synthetic division).

For example,

Factorise $x^3 + 2x^2 - x - 2$.

Find a linear factor. Remember from the Factor and Remainder theorem, if an x value is a root, then the function value is zero. The roots will be factors of the constant term (or if non-monic of the product of the leading coefficient and the constant – see example two below)

Try $x = 1$

$$1^3 + 2(1)^2 - 1 - 2 = 0$$

So we know $x = 1$ is a root and $x - 1$ is a factor.

Set up the division.

$$x - 1 \overline{)\, x^3 + 2x^2 - x - 2}$$

What do you need to multiply x by to get x^3? x^2

$$\begin{array}{l} \quad\quad x^2 \\ x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \quad\quad x^3 - x^2 \end{array}$$

Multiply $(x - 1)$ by x^2

Subtract

$$\begin{array}{l} \quad\quad x^2 \\ x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \quad\quad x^3 - x^2 \\ \quad\quad\quad\quad 3x^2 - x \end{array}$$

$x^3 + 2x^2 - (x^3 - x^2) = 3x^2$

What do you need to multiply x by to get $3x^2$? $3x$

$$\begin{array}{l} \quad\quad x^2 + 3x \\ x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \quad\quad x^3 - x^2 \\ \quad\quad\quad 3x^2 - x \\ \quad\quad\quad 3x^2 - 3x \end{array}$$

Multiply $(x - 1)$ by $3x$

Subtract

$$\begin{array}{l} \quad\quad x^2 + 3x \\ x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \quad\quad x^3 - x^2 \\ \quad\quad\quad 3x^2 - x \\ \quad\quad\quad 3x^2 - 3x \\ \quad\quad\quad\quad\quad 2x - 2 \end{array}$$

$(3x^2 - x) - (3x^2 - 3x) = 2x$

What do you need to multiply x by to get $2x$? $2x$

$$\begin{array}{l} \quad\quad x^2 + 3x + 2 \\ x - 1 \overline{)\, x^3 + 2x^2 - x - 2} \\ \quad\quad x^3 - x^2 \\ \quad\quad\quad 3x^2 - x \\ \quad\quad\quad 3x^2 - 3x \\ \quad\quad\quad\quad\quad 2x - 2 \\ \quad\quad\quad\quad\quad 2x - 2 \end{array}$$

Subtract

$$
\begin{array}{r}
x^2 + 3x + 2 \\
x - 1\overline{)x^3 + 2x^2 - x - 2} \\
x^3 - x^2 \qquad\qquad \\
3x^2 - x \qquad \\
3x^2 - 3x \qquad \\
2x - 2 \\
2x - 2 \\
0 \quad
\end{array}
$$

Factorise $x^2 + 3x + 2$

$x^2 + 3x + 2 = (x + 2)(x + 1)$

Hence $x^3 + 2x^2 - x - 2 = (x - 1)(x + 2)(x - 1)$

Example Two

Solve $2x^3 - x^2 - 13x - 6 = 0$

We need to factorise the cubic and then use the null factor law to solve it.

The roots will be multiples of -12 (2×-6), so $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$.

Let's try 2

$$2(2)^3 - 2^2 - 13(2) - 6 = 16 - 4 - 26 - 6 = -20$$

Try -2

$$2(-2)^3 - (-2)^2 - 13(-2) - 6 = -16 - 4 + 26 - 6 = 0$$

Hence, $(x + 2)$ is a factor.

$$
\begin{array}{r}
2x^2 - 5x - 3 \\
x + 2\overline{)2x^3 - x^2 - 13x - 6} \\
2x^3 + 4x^2 \qquad\qquad\quad \\
-5x^2 - 13x \qquad \\
-5x^2 - 10x \qquad \\
-3x - 6 \\
-3x - 6 \\
0 \quad
\end{array}
$$

$$
\begin{aligned}
2x^2 - 5x - 3 &= 2x^2 - 6x + x - 3 \\
&= 2x(x - 3) + 1(x - 3) \\
&= (2x + 1)(x - 3)
\end{aligned}
$$

$$2x^3 - x^2 - 13x - 6 = (x + 2)(2x + 1)(x - 3)$$

$$x = -\frac{1}{2} \quad x = 3 \quad x = -2$$